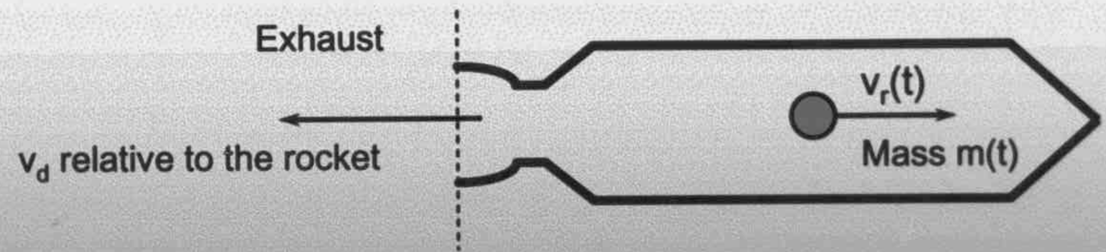
THE ROCKET EQUATION



Disregarding gravity forces, athmospheric pressure etc.:

No external forces means that the impulse is constant.

$$m \cdot v = const.$$

$$\frac{d}{dt}(mv) = 0 \qquad m \cdot \frac{dv}{dt} + v \cdot \frac{dm}{dt} = 0$$

$$dv = -v \cdot \frac{dm}{m}$$

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THRUST = skyv kraft

$$|\dot{P}_{rocket}| = |\dot{P}_{exhaust}|$$

$$|\dot{P}_{rocket}| = |\dot{m}\, v_{exit}|$$

$\dot{m}$ = mass flowrate of the exhaust products [kg/s]

v_{exit} = exit velocity of exhaust [m/s]

P_{rocket} = rocket's momentum [kg · m/s]

$\dot{P}_{rocket}$ = time derivative of —"— $\left[\frac{kg \cdot m}{s^2}\right]$ [N]

effective exhaust velocity could be slightly different from v_{exit}

$$F_{thrust} = \dot{m}\, v_{eff.exit}$$

v_{exit} (chemical rocket, Space Shuttle) ~ 3 km/s

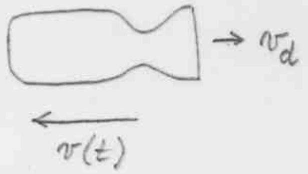
Power = energy expended per unit time.

Here: kinetic energy = $\frac{1}{2} m v_{eff.exit}^2$

$\Rightarrow$ power = $\frac{1}{2} \dot{m}\, v_{eff\text{-}exit}^2$. [J/s = W]

Space Shuttle $26.6 \cdot 10^9$ W !

THE ROCKET EQUATION



Newton's Second law

$$\sum F_{external} = \frac{d}{dt}(p) = \frac{d}{dt}(m \cdot v)$$

F: forces on object [N]
p: linear momentum [kg · m/s]
m: mass [kg]
v: velocity [m/s]

Here we have $\sum F_{external} = 0$

What is the momentum of the rocket?

$$p(t) = \underbrace{m(t) \cdot v(t)}_{\text{rocket itself}} + \int_0^t dm_d \, (v(t) - v_d)$$

sum of momentum combustion products from time zero to time t

$$\left(\approx \sum dm_d \cdot (v(t) - v_d) \right)$$

$$p(t) = m(t) \cdot v(t) - \int_0^t dm(t) \, (v(t) - v_d)$$

or $$p(t) = m(t) \cdot v(t) - \int_0^t \frac{dm(t)}{dt} (v(t) - v_d) \, dt$$

$$0 = \frac{dp}{dt} = \cancel{\frac{dm(t)}{dt}} \cdot v(t) + m \cdot \frac{dv(t)}{dt} - \cancel{\frac{dm(t)}{dt}} \cdot v(t) + \frac{dm(t)}{dt} \cdot v_d$$

$$\Rightarrow \quad m(t) \cdot \frac{dv(t)}{dt} = - \frac{dm(t)}{dt} \cdot v_d$$

$$\frac{dv(t)}{dt} = - \frac{dm(t)}{dt} \cdot \frac{1}{m(t)} \cdot v_d$$

$$\int_{v_i}^{v_f} dv(t) = - \int_{m_i}^{m_f} \frac{dm(t)}{m(t)} \cdot v_d$$

$$v_f - v_i = \Delta v = - v_d \cdot \ln m \Big]_{m_i}^{m_f}$$

$$\therefore \quad v_f - v_i = + v_d \cdot \ln\left(\frac{m_i}{m_f}\right)$$

$v_f - v_0$ = velocity change [m/s]

v_d = effective exhaust velocity [m/s]

m_i = initial vehicle mass, before firing [kg]

m_f = final vehicle mass, after firing [kg]

Specific impulse

total impulse I

$$I \equiv F \cdot \Delta t = \Delta p$$

I : total impulse [Ns]
F : force [N]
Δt : time [s]
Δp : momentum change [N·s]

To compare different types of rockets we use the specific impulse, I_{sp}

$$I_{sp} = \frac{I}{\Delta m_{propellant} \cdot g_0}$$

I : total impulse [Ns]
Δm : change in propellant mass [kg]
g_0 : gravitational constant [m/s²]

$$I_{sp} = \frac{F_{thrust} \cdot \Delta t}{\Delta m_{prop} \cdot g_0} \qquad [I_{sp}] = s$$

$$= \frac{F_{thrust}}{\dot{m} \cdot g_0} = \frac{\dot{m}\ v_{eff \cdot exit}}{\dot{m}\ g_0}$$

$$\therefore \quad \boxed{I_{sp} = \frac{v_{eff-exit}}{g_0}}$$

We have

$$v_f - v_i = \Delta v = v_{exit} \log\left(\frac{m_r + m_d}{m_r}\right)$$

m_r : mass rocket [kg]

v_{exit} : efficient exit velocity [m/s]

m_d : mass of propellant [kg]

or

$$e^{\frac{\Delta v}{v_{eff.exit}}} = e^{\log\left(\frac{m_r + m_d}{m_r}\right)}$$

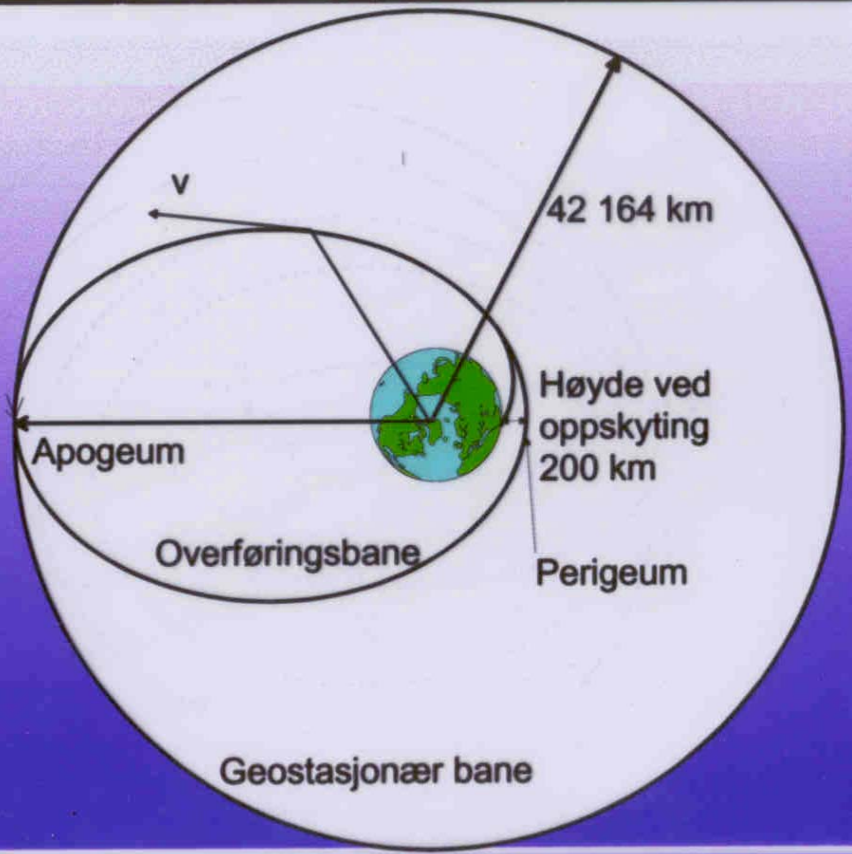
$$\Rightarrow \frac{m_r + m_d}{m_r} = e^{\Delta v / v_{eff.exit}}$$

$$m_d = m_r \left(e^{\frac{\Delta v}{v_{eff.exit}}} - 1\right)$$

or

$$\boxed{m_d = m_r \left(e^{\frac{\Delta v}{I_{sp} \cdot g_0}} - 1\right)}$$

Overføring til geostasjonær bane



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Orbit Adjustment

It was noted earlier that if the launch vehicle does not establish the desired orbital position and velocity precisely, or if the initial orbit is not the spacecraft's operating orbit, then an orbit adjustment will be required. We differentiate orbital adjustments from station keeping or attitude adjustments by the relatively larger Δv requirements of orbital adjustments, as quantified in Chapter 2. Missions requiring orbital adjustments may incorporate completely separate propulsion systems to accomplish these maneuvers, while station keeping and attitude adjustment (if performed by a propulsion system) may be accomplished by a system integrated into the design of the spacecraft.

Addressing all possible types of orbital adjustments and their associated propulsion requirements is beyond the intent of this book. However, an excellent and commonly performed example exists which can be used to describe both the magnitude of adjustment required and the typical types of systems used to perform these maneuvers.

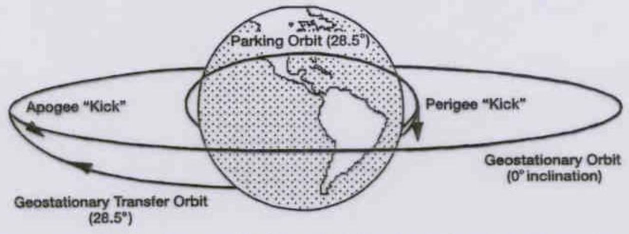


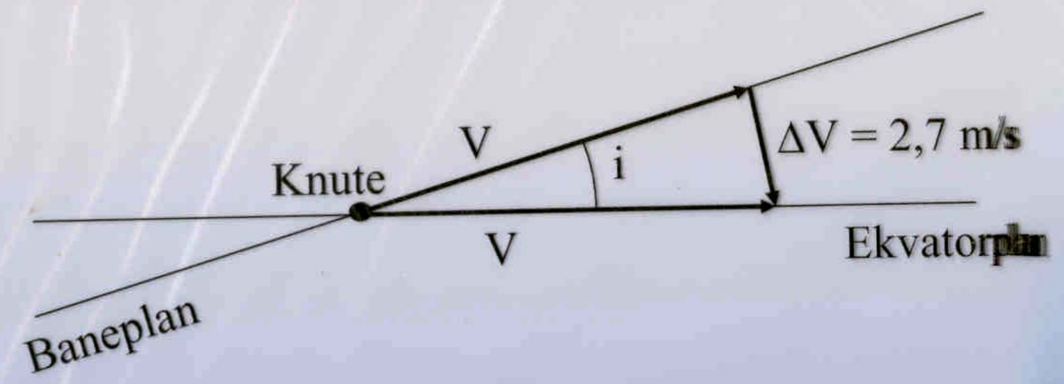
Figure 3-9. Geostationary orbital transfer. The plane-change maneuver is performed out at the apogee of the transfer orbit to minimize the Δv required.

Geostationary Transfer. Figure 3-9 illustrates the transfer of a satellite from a typical low-earth "parking" orbit of 28.5° inclination to a geostationary orbit.

The first maneuver establishes the spacecraft on a *transfer orbit* which is an ellipse that lies in the same plane as the parking orbit with its perigee at the same altitude as the parking orbit and its apogee at geostationary altitude. This should be recognized as a Hohmann transfer orbit described in Chapter 2. Note that in order to intersect the final desired equatorial geostationary orbit, the apogee (and, thus, the perigee) of the 28.5° inclined transfer orbit must be located over the equator. As the satellite approaches this point over the equator, a *perigee kick motor* (PKM) is fired to inject the spacecraft into the transfer orbit from the parking orbit. The change in velocity required at this point can be found from the relationships given in Chapter 2.

At the transfer orbit apogee, two changes to the orbit must occur in order to establish a geostationary orbit. First, the elliptical transfer orbit must be circularized at the geostationary altitude. If this is done as a separate maneuver within the same plane as the transfer orbit, the Δv required would be the same as for a Hohmann transfer. However, to be geostationary, the orbit must also be equatorial, so the plane of our spacecraft orbit must be changed from 28.5° to 0°. The Δv relationship for a simple plane,

Inklinasjonskontroll



$$m_p = m_s \cdot (e^{\Delta V / I_{sp} \cdot g} - 1) = 1\ kg$$

$m_s = 800 kg$ $\qquad I_{sp} = 230 sek$

$V = 3{,}075 km/s$ $\qquad i = 0{,}05^0$

Rocket Thrust

Looking more closely at the results of the last section, it can be seen that the thrust term of the rocket equation is proportional to both the propellant exhaust velocity ($\overline{v} - \overline{v}_o = \overline{v}_e$) and the *mass flow rate of propellant* (**dm/dt** = **ṁ**). We can rewrite the thrust term to better show this dependency:

$$\overline{T} = (\overline{v} - \overline{v}_0)\frac{dm}{dt} = \overline{v}_e\dot{m} \qquad (3\text{-}4)$$

To increase the thrust of a rocket then, one could try to increase either the exhaust velocity of the propellant or the mass flow rate of propellant through the rocket. To see how these quantities can be changed we must consider the characteristics of a typical rocket system such as that shown in Figure 3-2.

Mass Flow Rate. A rocket differs from a jet engine in that a rocket must carry its own oxidizer as well as fuel supply, although there are some systems which simply use a single (mono) propellant. In many propulsion systems, liquid propellants are used and are delivered to the combustion chamber by mechanical pumps as depicted in the figure. The pumps contribute directly to the rocket thrust by controlling the mass flow rate of the propellant. Lower-thrust systems may use a pressurized bladder or gravity-feed to deliver propellants to the combustion chamber. In solid rocket motors, the solid fuel and oxidizer materials are premixed and loaded into the motor casing which also serves as the combustion chamber when the fuels are ignited. Mass flow rate in solids is established by controlling the burning area and, thus, the combustion rate of the fuel.

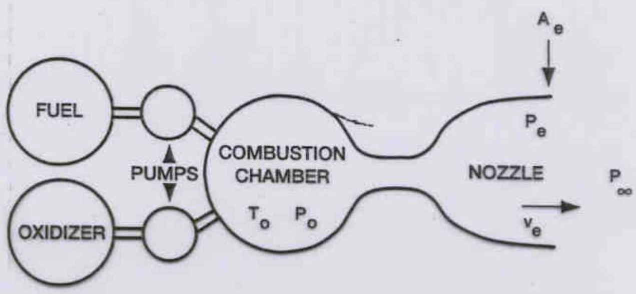


Figure 3-2. Sketch of a typical rocket. Propellants are mixed in the combustion chamber and accelerated through the nozzle to produce thrust.

There are some obvious limitations to increasing thrust by increasing the mass flow rate. For instance, you could burn all the fuel at once, assuming you could design a combustion chamber and nozzle to handle the amount of propellant and exhaust involved. However, if you didn't just blow up, the instantaneous thrust would probably produce an unacceptable acceleration in the view of the structural designer or payload/ astronaut. More practically, the size and structural capabilities of the pumps in liquid-fueled systems limit the mass flow rates achievable. Pumps also contribute to thrust by affecting the exhaust velocity, as discussed next.

Bernoulli (1700-1782)

$$h + \frac{1}{2} v^2 = constant$$

or

$$Pv + u + \frac{1}{2} v^2 = constant$$

($\frac{1}{2} v^2$ is the specific kinetic energy of flow)

In a rocket engine we have a combustion chamber and a nozzle

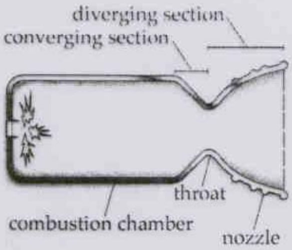


Figure 14-14. Standard Combustion Chamber and Nozzle Configuration. A standard thermodynamic rocket has two main parts—a combustion chamber (where energy transfers to a propellant) and the nozzle (where high energy combustion products convert to high-velocity exhaust).

From the combustion chamber the products flow into the converging section of nozzle. The gas velocity increases (since A decreases, v must increase to keep $\dot{m}$ constant.

At the throat the velocity is roughly equal to the speed of sound, $M_a = 1$ (Mach number)

The Rocket Equation (cont'd)

Integrating and inserting the mass before ignition, $m_0 + m_d$ and mass after burning, m_0, gives the velocity increase Δv

$$\Delta v = v_d \int_{(m_0+m_d)}^{m_0} \frac{dm}{m} = v_d \ln\left(\frac{m_0 + m_d}{m_0}\right)$$

It is also possible to express the equation on exponential form

$$\frac{m_0 + m_d}{m_0} = e^{\left(\frac{\Delta v}{v_d}\right)}$$

$$m_d = m_0 \cdot (e^{\left(\frac{\Delta v}{v_d}\right)} - 1)$$

The exhaust velocity characterized by its specific impulse $v_d = I_{sp} \times g$

I_{sp} = specific impulse (sec), g is the acceleration of the earth's gravitational field

TO REACH ABOUT 8000 m/s

$$\Delta v = v_{eff.exit} \log\left(\frac{m_r + m_d}{m_r}\right)$$

$$m_r = K\,(m_d + m_s)$$

↳ rocket ↳ propellant ⟶ payload

$$e^{\frac{\Delta v}{v_{eff.exit}}} = \frac{m_r + m_d}{m_r} = \frac{K m_d + K m_s + m_d}{K m_d + K m_s}$$

$$\frac{m_d}{m_s} = \frac{e^{\frac{\Delta v}{v_{eff.exit}}} - 1}{1 - \frac{K}{K+1}\, e^{\frac{\Delta v}{v_{eff.exit}}}}$$

denominator $\rightarrow 0$ means that $m_s \rightarrow 0$

This is the case for $V_{eff.exit} \sim 3\,km/s$, $K = 0.1$

$\Rightarrow \Delta v = 7.2\ km/s$!!

Necessary with <u>more than one stage</u>

NOZZLES

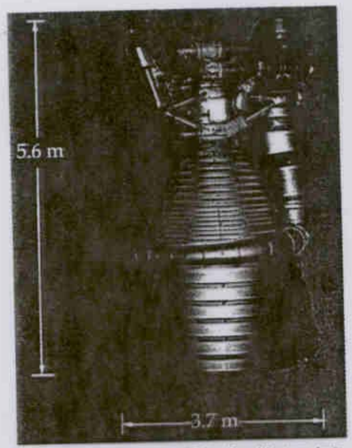


Figure 14-9. Saturn V Nozzles. Most rockets rely on nozzles to convert thermal energy into kinetic energy through thermodynamic expansion. We show the huge nozzles for the Saturn V F-1 engines here. *(Courtesy of NASA/ Marshall Space Flight Center)*

Total energy in propellant.

Specific enthalpy $h = pV + u$

h: specific enthalphy $[J/kg]$
p: pressure $[N/m^2]$
V: volume, specific $[m^3/kg]$
u: internal energy $[J/kg]$

mass flow rate: let the gas pass a cross sectional area A:

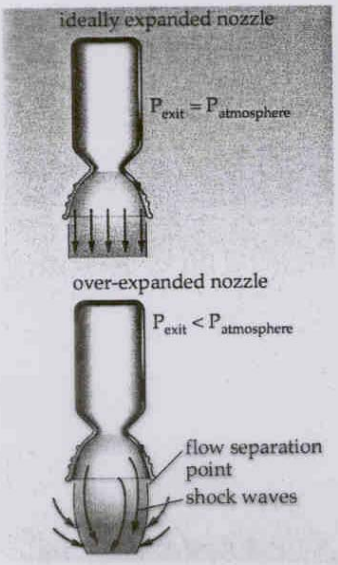
$$\dot{m} = A \cdot v \cdot \rho$$

$\dot{m}$: mass flow rate $[kg/s]$
A: cross-sectional area $[m^2]$
v: fluid's velocity $[m/s]$
ρ: fluid's density $[kg/m^3]$

The nozzle's expansion ratio ε

$$\varepsilon = \frac{A_{nozzle's\ exit}}{A_{throat}}$$

affects the engine performance, as does the pressure inside and outside nozzle.



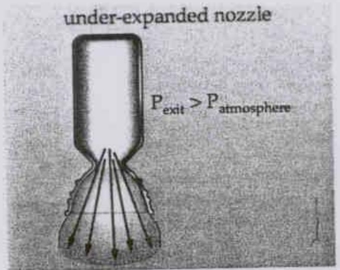


Figure 14-16. Nozzle Expansion. To effectively convert all the enthalpy available in the combustion products to high-velocity flow, we need the nozzle exit pressure (P_{exit}) to equal the outside atmospheric pressure ($P_{atmosphere}$). When $P_{exit} < P_{atmosphere}$, the flow is overexpanded, causing shock waves that decrease flow velocity. When $P_{exit} > P_{atmosphere}$, the flow is underexpanded meaning not all available enthalpy converts to velocity. Here, we show all three expansion cases. In practice, we need an infinitely long nozzle to achieve perfect expansion in a vacuum.

For velocities corresponding to $M_a > 1$, thermodynamics predicts:

$$\frac{dA}{A} = (M_a^2 - 1)\frac{dv}{v}$$

dA: infinitesimal area change $[m^2]$
A: area of cross section $[m^2]$
M_a: Mach number $\left(\frac{v}{v_0} = \frac{v}{\text{speed of sound}}\right)$
dv: infinitesimal velocity change $[m/s]$
v: velocity change $[m/s]$

How curious!
For low speeds $M_a < 1$ and a smaller cross sectional area leads to higher speed v.
For speeds $M_a > 1$ a larger cross sectional area leads to higher speeds. v!

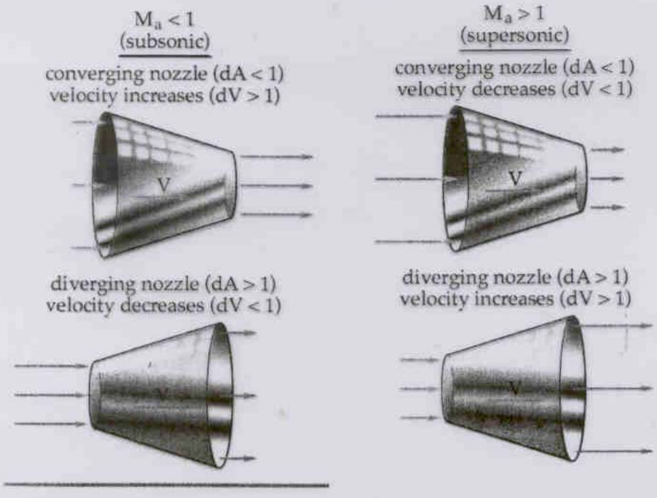


Figure 14-13. Changing Mach Num Versus Changing Area. For subsonic ($M_a < 1$) the flow velocity increases when a decreases along the nozzle. For supers flow ($M_a > 1$) the flow velocity increases w the area increases.

ORBIT ESTABLISHMENT AND ORBITAL MANEUVERS

As was mentioned at the end of Chapter 2, a spacecraft's propulsion requirements may include delivery to space, maneuvering into position, and maintenance of the spacecraft position and orientation. While position-keeping systems are usually designed as an integral part of the space-

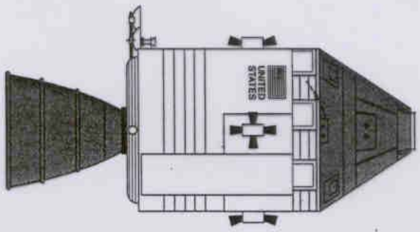


Figure 3-5. Apollo Service Module propulsion systems. The large aft nozzle was used for orbital maneuvers and the smaller nozzles around the body were used for attitude control.

craft, rockets that deliver a spacecraft to its operating position are more likely to be existing systems which are chosen for specific performance characteristics and to which the spacecraft must be designed for compatibility. Specific systems that are available to meet these requirements will be presented in Chapter 8. The following sections describe the propulsion aspects that must be taken into account when discussing these systems.

Orbit Establishment

Several factors are involved in reaching the desired orbit, including establishing the correct burn-out conditions, staging, launch timing, launch pad location, and launch direction. Figure 3-6 depicts a typical launch situation from the U.S. launch site at the Kennedy Space Center (KSC) in Florida.

Burn-out Conditions. The launch vehicle both accelerates and raises the payload from the surface of the earth along a predetermined launch path (trajectory). The launch path must intersect the desired orbit, and the intersection must occur such that the rocket velocity (vector) and altitude correspond with those for the desired orbit *at that point.* Since the rockets cease burning fuel at this point, the velocity and altitude achieved are known as the launch vehicle *burn-out* conditions. If the burn-out conditions do not match the orbit characteristics at that point, within some allowable tolerance, adjustments using supplemental propulsion systems may be required.

Example 14-3

Problem Statement

Imagine you are preparing the new Falcon launch vehicle for its first mission from Kennedy Space Center. The vehicle must deliver a total ΔV (ΔV_{design}) of 10,000 m/s. The total mass of the second stage, including structure and propellant, is 12,000 kg, 9000 kg of which is propellant. The payload mass is 2000 kg. The I_{sp} of the first stage is 350 seconds and of the second stage is 400 seconds. The structural mass of the first stage is 8000 kg. What mass of propellant must be loaded on the first stage to achieve the required ΔV_{design}? What is the vehicle's total mass at lift-off?

Problem Summary

Given: 2 stages

$m_{payload} = 2000$ kg

$m_{structure-2} + m_{propellant-2} = 12{,}000$ kg

$m_{propellant-2} = 9000$ kg

$m_{structure-1} = 8000$ kg

$I_{sp-1} = 350$ s

$I_{sp-2} = 400$ s

$\Delta V_{design} = 10{,}000$ m/s

Find: $m_{propellant-1}$

$m_{initial}$

Conceptual Solution

1) Determine the $\Delta V_{stage\ 2}$

$$\Delta V_{stage\ 2} = I_{sp\ 2} g_o \times \ln\left(\frac{m_{structure\ 2} + m_{propellant\ 2} + m_{payload}}{m_{structure\ 2} + m_{payload}}\right)$$

2) Determine the required ΔV of stage 1

$$\Delta V_{stage\ 1} = \Delta V_{design} - \Delta V_{stage\ 2}$$

3) Determine the initial mass of stage 1

$$\Delta V_{stage\ 1} = I_{sp\ 1} g_o \times \ln\left(\frac{m_{initial}}{m_{structure\ 2} + m_{propellant\ 2} + m_{payload} + m_{structure\ 1}}\right)$$

4) Determine the mass propellant in stage 1

$$m_{propellant\ 1} = m_{initial} - (m_{stucture\ 1} + m_{structure\ 2} + m_{propellant\ 2} + m_{payload})$$

Analytical Solution

1) Determine

$$\Delta V_{stage\ 2} = I_{sp\ 2} g_o \times \ln\left(\frac{m_{structure\ 2} + m_{propellant\ 2} + m_{payload}}{m_{stucture\ 2} + m_{payload}}\right)$$

$$= (400\ s)(9.81 m/s^2)\ \ln\left(\frac{12,000 kg + 2000 kg}{3000 kg + 2000 kg}\right)$$

$$\Delta V_{stage\ 2} = 4040\ m/s$$

2) Determine the required ΔV of the first stage

$$\Delta V_{stage\ 1} = \Delta V_{design} - \Delta V_{stage\ 2}$$

$$= 10{,}000\ m/s - 4040\ m/s$$

$$\Delta V_{stage\ 1} = 5960\ m/s$$

3) Determine the initial mass of stage 1

$$\Delta V_{stage\ 1} = I_{sp\ 1} g_o \times \ln\left(\frac{m_{initial}}{m_{structure\ 1} + m_{structure\ 2} + m_{propellant\ 2} + m_{payload}}\right)$$

$$m_{inititial} = (8000\ kg + 3000\ kg + 9000\ kg + 2000\ kg)\, e\left[\frac{5960 m/s}{(-350 s)(9.81 m/s^2)}\right]$$

$$m_{initial} = 124{,}821\ kg$$

4) Determine mass of propellant in stage 1

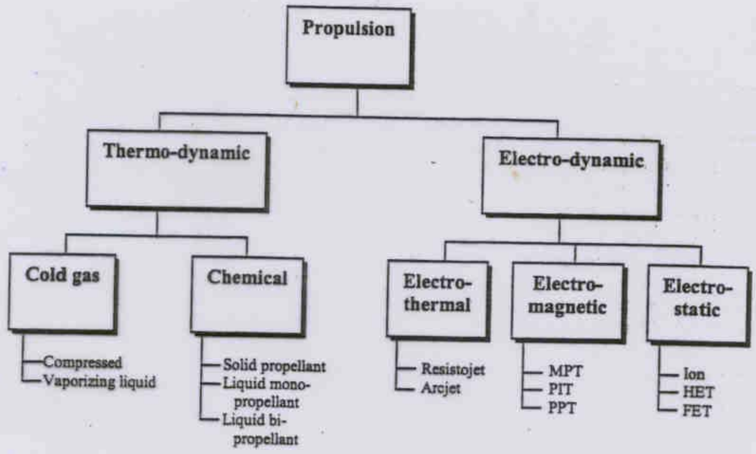
$$m_{propellant\ 1} = m_{initial} - (m_{structure\ 1} + m_{structure\ 2} + m_{propellant\ 2} + m_{payload})$$

$$= 124{,}821 - (8000\ kg + 3000\ kg + 9000\ kg + 2000 kg)$$

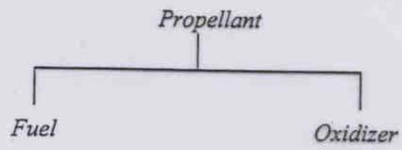
$$m_{propellant-1} = 102{,}821\ kg$$

Interpreting the Results

The total mass of this launch vehicle at lift-off is 124,821 kg (113 tons). About 82% of this mass is propellant in the first stage alone (102,821 kg/124,821 kg). Less than 2% of the total lift-off mass is payload (2000 kg/124,821 kg).



Propulsion technologies for satellite applications.



The most important figure of merit of a particular propellant is its specific impulse I_{sp}, defined as follows:

$$I_{sp} = \frac{V_e}{g} = \frac{F}{g\,\dot{m}} = \frac{F\,\Delta t}{g\,\Delta m} \qquad \text{(seconds)} \qquad (6.1)$$

In Fig 6.3, the fuel and the oxidizer are pressurized by helium. The two components are fed separately to the combustion chambers of the apogee kick engine (AKE) and the thrusters, where they ignite spontaneously. The thrusters are used in various combinations for orbit control as well as attitude control. Only one set of thrusters is used, the other set being redundant.

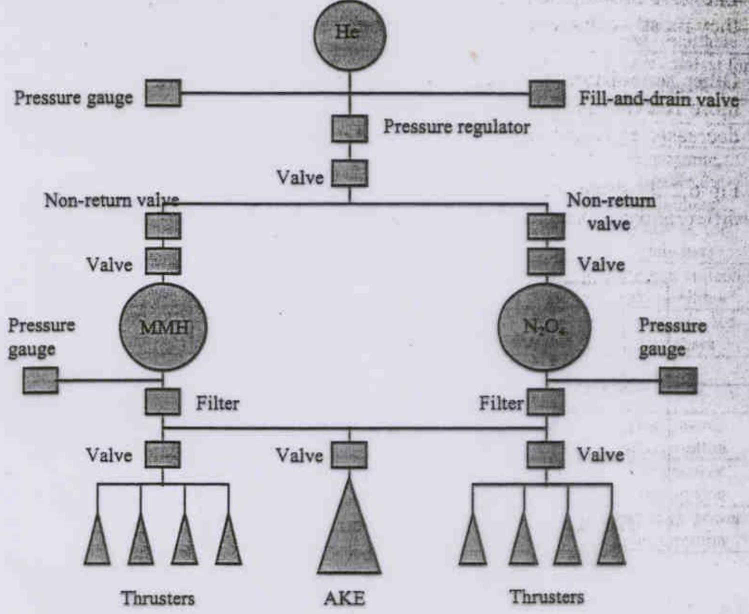


Fig 6.3: Bipropellant propulsion architecture for a three-axis stabilized satellite.

Propellant Storage

Liquid Propellants

Feeding liquid propellants from satellite storage tanks is complicated by the weightlessness of space. Where spin-stabilized satellites are concerned, the centrifugal force creates an artificial gravity which is exploited as shown to the left in Fig 6.5.

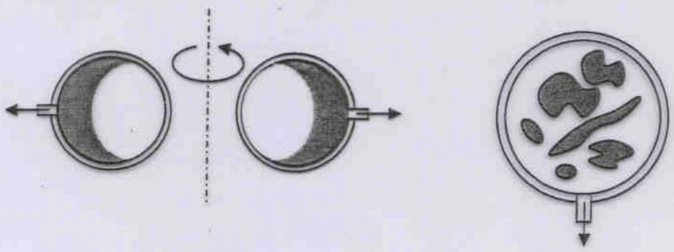


Fig 6.5: Propellant feed with and without centrifugal force.

The figure to the right illustrates the dilemma with a body-stabilized spacecraft, where the liquid's surface tension causes it to form lumps that hover inside the tank in an uncontrolled manner. Pressurizing the tank does not solve the problem, since the pressurant fills all empty spaces equally.

There are two common methods of forcing the propellant into the drain (Fig 6.6). One is to install an elastic membrane which acts as a bladder. This solution works reasonably well until the membrane is flat and the remaining propellant is trapped.

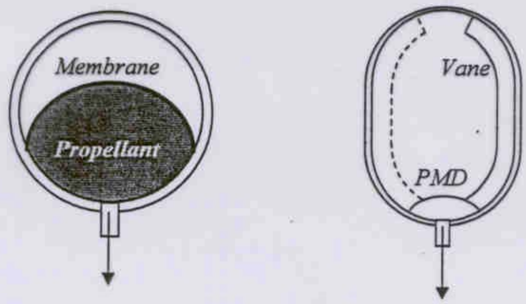


Fig 6.6: Propellant management using bladder and vanes.

The second method allows all the propellant to be drained in weightlessness. Vanes are attached to the inside wall of the tank. Surface tension causes the liquid to adhere to the vanes and work its way down to the *propellant management device* (PMD) at one end of the tank. The PMD is basically a piece of wire mesh shaped like a clamshell, which allows liquid but not gas to filter through.

The membrane solution works well with monopropellant hydrazine but is prone to corrosion damage by the oxidizer in bipropellant hydrazine systems. The PMD is therefore the norm for the MMH + N_2O_4 combination.

Solid Propellants

Fig 6.7 shows a cross-section of a typical solid propellant AKM.

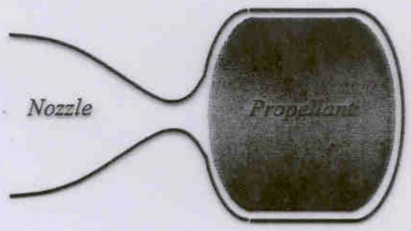


Fig 6.7: Solid propellant apogee kick motor.

Before a solid propellant AKM is filled, the inside wall of the casing is coated with a rubber-like liner which improves the adhesion of propellant to the wall and serves as thermal insulation. The propellant is cast such that a hollow core results. The hollow provides a larger burning surface, and hence greater thrust, than a solid casting. The shape of the hollow determines the thrust profile, as illustrated in Fig 6.8.

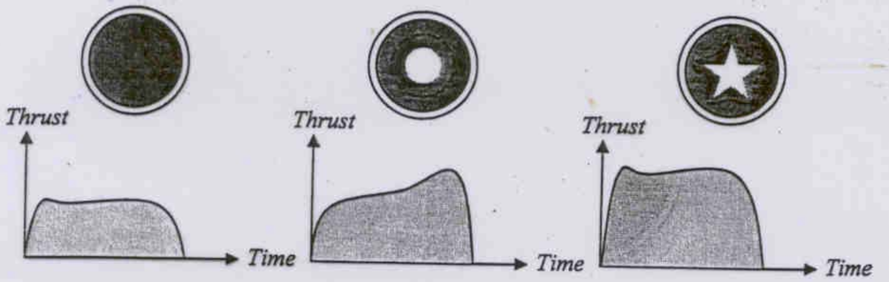


Fig 6.8: Thrust profiles as a function of core geometry, as seen from the nozzle end.

A solid casting will give approximately constant thrust, which is preferred in most missions (see left figure), but the exposed surface is too small to yield adequate thrust level. By drilling or casting a cylindrical hole through the centre, we achieve a larger burning area and therefore greater thrust. But as the combustion erodes the walls of the hole, the burning area becomes larger, and the thrust grows with time (see dotted contour of the middle figure). This may or may not be a desirable side effect.

Electrodynamic Propulsion (EP)

Several electrodynamic thruster technologies have been developed over the last 20 years, inspired by the promise of extremely low propellant consumption for a given total impulse compared to the thermodynamic alternatives.

There are three classes of electric thrusters: *electrothermal, electrostatic* and *electromagnetic*. In the electrothermal variety, thrust is generated by electrically super-heating neutral gases. In electrostatic and electromagnetic thrusters, a neutral gas or solid is electrically ionized, and thrust is created by accelerating the ions by means of an electrostatic or electromagnetic field.

Each type of EP thruster is divided into sub-groups depending on the adopted design solution. Table 6.2 summarizes the main characteristics of the various solutions. Not all of these are, or ever will be, suitable for earth-orbiting satellites. They are nevertheless included for the sake of completeness.

Electrostatic Thrusters

The **ion thruster** is a "textbook ionizer." The neutral propellant molecules (e.g. xenon gas) are stripped of outer layer electrons through electron bombardment (Fig 6.14). The resulting positive ions are accelerated by a grid-induced electrostatic field. Following expulsion, the ions are blended with neutralizing electrons to prevent them from settling on the spacecraft's surface. In the absence of a neutralizer, the returning ions would create a voltage differential, which might eventually give rise to electrostatic discharge (Chapter 12). Several other ion thruster technologies are at various stages of development, notably the **contact ion engine, microwave ion engine, radio frequency ion engine, plasma separator ion engine,** and **radio isotope ion engine.**

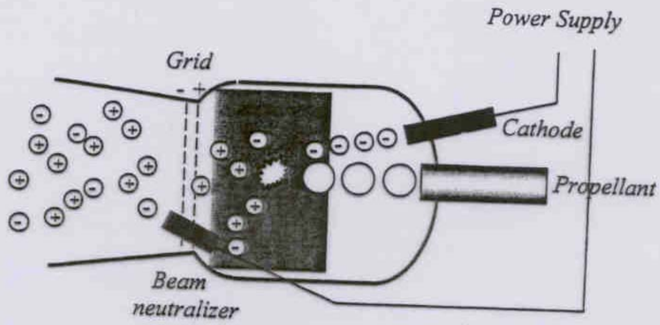


Fig 6.14: Schematic of an ion thruster.

6.4 Thruster Layout

A satellite requires two sets of thrusters, one for orbit control and the other for attitude control. Since orbit control means imparting a translation movement, the corresponding thrusters must be mounted and aligned in such a way that their combined thrust vectors act on the satellite's centre of mass. Attitude control, on the other hand, involves rotation movements, so the corresponding thrust vectors should act as far away as possible from the centre of mass to achieve maximum moment (Fig 6.16).

	I_{sp} in vacuum	Thrust (N)	Handling hazard	Ease of storability	Comment
Cold gas	75	0.1 - 250	Low	Very easy	Inexpensive
Monopropellants					
Solid propellants	250 – 290	100 - 10^6	High	Easy	Not restartable
Hydrazine	200 - 250	< 1	High	Easy	Needs catalyst for combustion
Bipropellants					
Kerosene + O_2	300 - 350	10 - 10^6	Very high	Difficult	Cryogenic, suitable for launchers
UDMH + N_2O_4	300 - 350	10 - 10^6	High	Difficult	Storable, for launchers
MMH + N_2O_4	300 - 350	0.1 - 500	High	Difficult	Storable, suitable for satellites
H_2 + O_2	440 - 460	10 - 10^6	Very high	Very difficult	Cryogenic, for launchers
Electric Propulsion					
Electrothermal	< 1500	< 2	Very low	Very easy	High power consumption
Electrostatic	< 6000	< 1	Very low	Very easy	High power consumption
Electromagnetic	< 5000	< 200	Very low	Very easy	Very high power consumption

Table 6.1: Comparison between various types of propellant.

From the Table it is evident that no single propellant is the perfect choice for all applications. For example, the low thrust of electric propulsion (< 2 N) restricts its use to manoeuvres that are not urgent, such as slow flywheel momentum dumping or GEO station-keeping. As for bipropellants, the fuel and the oxidizer have to be kept well apart during launch preparations, as they are *hypergolic*, i.e. they combust on contact with each other. Liquid oxygen and hydrogen are not only hypergolic, but they are also *cryogenic*, i.e. they must be kept refrigerated below their vapour temperatures – all of which makes for difficult storage and handling. Liquid monopropellants are easier to store and handle, because the fuel and the oxidizer are contained in the propellant itself. Solid monopropellants are very reliable but have the great disadvantage of not being re-startable, which precludes stepwise execution of orbital manoeuvres.

As indicated in Table 6.1, the high thrust of UDMH makes it most suitable for launch vehicles. H_2 and O_2 must be stored in liquid form at cryogenic temperatures, so the kerosene+O_2 and H_2+O_2 alternatives are also reserved for launchers. Solid propellants are most commonly found on launch vehicles, but sometimes this technology is chosen for apogee kick motors (AKM) onboard satellites. This is hardly surprising, since the AKM actually performs the task of the launch vehicle.

Category	Propellant choices	Vacuum I_{sp} (s)	Thrust (N)	Power/thrust conversion efficiency (%)	Power per thruster (W)	Development status
Electrothermal						
Resistojet	N_2, H_2, N_2H_4, NH_3	300-1000	< 0.5	60 - 90	> 300	Very mature
Arcjet	H_2, N_2H_4, NH_3	700-1500	< 2	30	> 300	Mature
Electrostatic						
Ion	Ar, Cs, Hg, Xe	< 5000	< 1	90	500-5000	Mature
HET	Xe	< 2500	< 1	60	1500	Mature
FET	Cs	< 6000	< 1	50 - 60	20 - 120	Mature
Electromagnetic						
MPD	Ar, Ne, Xe, NH_3, N_2H_4, H_2, N_2, Li, K, Na	1000-4000	10 - 200	10 - 40	Several hundred kW	Immature
PIT	N_2H_4, NH_3, CO_2, Ar	3000-5000	1 - 150	20 - 40	Several kW	Immature
PPT	Teflon	800-1200	< 1	< 20	200 - 400	Fairly mature

Table 6.2: Overview of electric propulsion thrusters.
HET = Hall Effect Thruster
FET = Field Effect Thruster
MPD = Magnetoplasmadynamic
PIT = Pulsed Inductive Thruster
PPT = Pulsed Plasma Thruster

As mentioned earlier, the main advantage of electric propulsion (EP) over chemical ditto is the high I_{sp}, which means that a substantially greater impulse I_t can be generated for a given propellant mass m (Eq 6.2). Worded differently, the thrust-to-propellant consumption ratio is more favourable. The high I_{sp} is the direct result of the increase in the exit velocity V_e of EP molecules (> 10 km/s) compared to chemical thrusters (~ 3 km/s) – see Eq 6.1. In EP, the exit velocity is proportional to the applied voltage and, at least in theory, only the sky is the limit. In chemical propulsion, on the other hand, the reactants have a finite amount of energy per unit mass, which ultimately limits the achievable exit velocity.

The kinetic energy of a propellant ion is equal to the work done by the electric field that accelerates it. Thus:

$$\tfrac{1}{2}mV_e^2 = qU \qquad (6.4)$$

where m is the mass, V_e is the exit velocity and q is the charge of the ion, while U is the accelerating voltage.

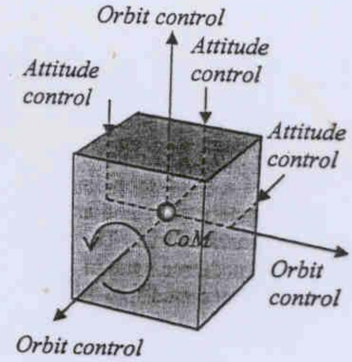


Fig 6.16: Thruster placement for attitude and orbit control.

Spin stabilization

Spin-stabilized satellites offer limited scope for thruster placement. A GEO spinner is a good example (Fig 6.17). The satellite's spin axis is aligned with that of the earth. Three types of manoeuvre must be supported: N/S station-keeping, E/W station-keeping, attitude control and spin rate control.

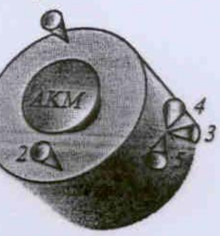


Fig 6.17: Thruster layout on a spin-stabilized GEO satellite.

N/S station-keeping requires a thrust along the spin axis through the centre of mass. Ideally, this manoeuvre should be performed by a single thruster mounted in the centre of an end panel. However, the end surfaces may already be occupied by an AKM (as in the picture) or by a despun antenna. The alternative is therefore to mount two thrusters (No. 1 and 2) along the periphery 180° apart, and to fire them continuously for as long as it takes to achieve the desired ΔV.

Body stabilization

For full attitude control, the attitude control thrusters must be mounted in such a manner that the satellite may be rotated in both the left-hand and the right-hand direction around the axes *x*, *y* and *z*.

Similarly, for complete orbit control, the satellite must be able to be propelled along the axes *x*, *y* and *z*, as well as in the opposite directions -*x*, -*y* and -*z*.

The thruster layout in Fig 6.18 would achieve this, as listed in the table.

Attitude control (rotation) axis	Thruster No.
x	4
y	1
z	9
-x	6
-y	3
-z	7
Orbit control (translation) axis	**Thruster No.**
x	10
y	11
z	12
-x	8
-y	5
-z	2

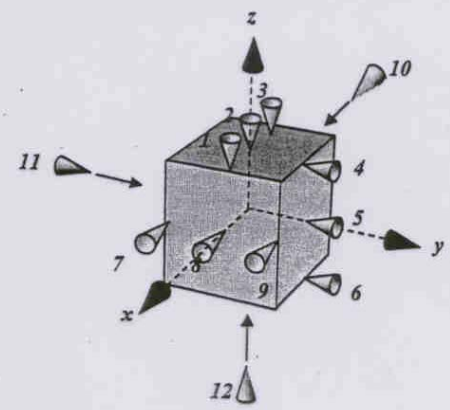


Fig 6.18: Thruster layout for 12 degrees of freedom.

This layout implies that 12 thrusters are needed, to which must be added another 12 thrusters for redundancy, i.e. 24 thrusters in total. As we shall see, such design profligacy is neither acceptable nor necessary.

For starters, let us examine what is needed for orbit control. We can do away with thrusters 2, 5 and 8, since their functions can be performed by simultaneous firing of the thruster pairs 1+3, 4+6 and 7+9, respectively. We are down to 9 thrusters (plus 9 redundant ones).

Assume that the satellite's positive *x*-axis is parallel to the velocity vector at some point along the orbit. If the orbit is circular, then the negative *x*-axis points along the velocity vector at a true anomaly 180° away from the first point (Fig 6.19). Therefore we do not require both thrusters 8 and 10, since one of them is enough to either speed up or slow down the satellite. So we eliminate thruster 10, and we have already replaced 8 through simultaneous operation of 7 + 9. We are now down to 8 thrusters (plus 8 for redundancy). Perhaps we can then eliminate the *z*-pointing thrusters 2 and 12 altogether, since they, too, point along the velocity vector at half-orbit intervals. But we already replaced thruster 2 with 1 + 3, so if 12 goes, we are left with 7 operational thrusters (plus 7 spares).

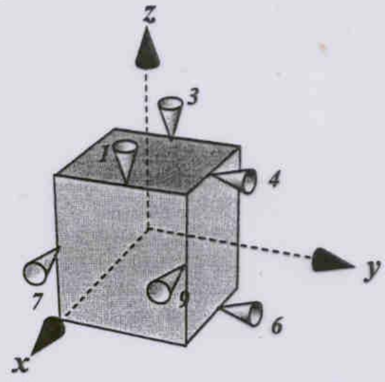


Fig 6.20: Hypothetical thruster layout after savings.

Performance Summary

Fig 6.21 offers a graphical overview of the performance and utilisation of various propulsion technologies.

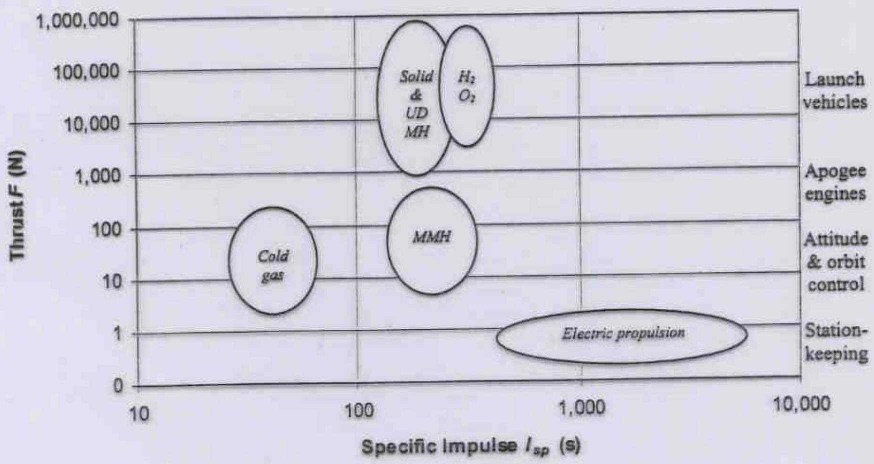


Fig 6.21: Thrust versus specific impulse for various propellants.

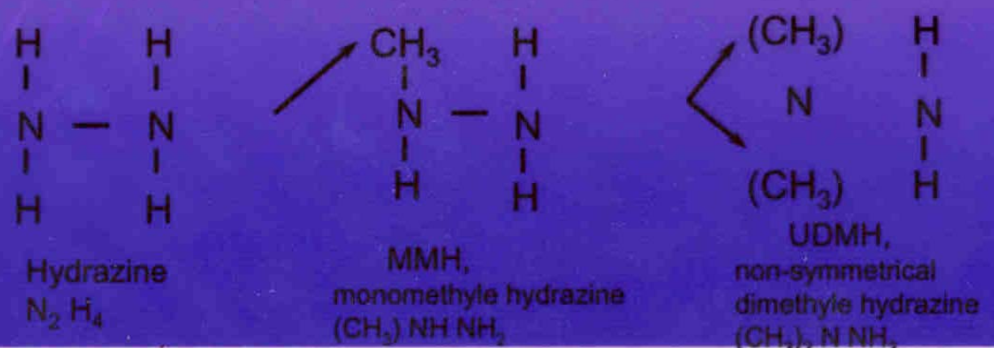
TYPES OF ROCKET ENGINES

- Solid propellant
  - fuel + oxydator + binding substance
  - pyrotechnical ignitors
  - simple construction, reliable, high thrust
  - cannot be stopped and restarted
- Liquid propellant
  - monopropellant stored in a single tank
    - exothermic decomposition of a liquid, such as Hydrazin
  - bipropellant, stored in separate tanks
    - hypergolic if the react spontaneously upon contact
    - Commonly used bipropellant fuels are MMH and UDMH
- Characterized by specific inpulse
  - $I_{sp} = g \times v_d$ where $g = 9.81\ m/s^2$
  - v_d is the exhaust velocity

PROPELLANTS

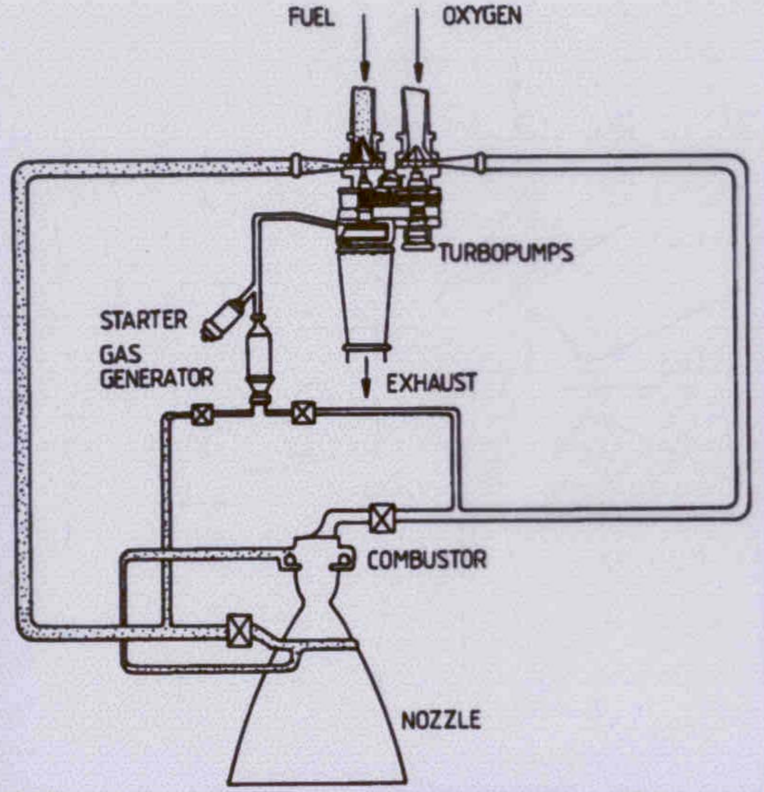
Single component fuel	Typical I_{sp}(s)
Cryogenic nitrogen	75
Solid fuel	210 - 290
Hydrazine N_2H_4	220 - 300

Two component fuel	
UDMH + N_2O_2	240 - 290
MMH + N_2O_4	300 - 315
H_2 + O_2	440 - 460



NTNU

Rocket engine for two component propellant



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